Sec. 11.2 Polynomial Functions

Polynomial Function – a sum of power functions whose exponents are nonnegative integers or a function in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$. The domain is the set of all real numbers.

a. Polynomials where f(x) = 0 are called the zero polynomial but do NOT have a degree.
b. Polynomials where f(x) = # are called constant polynomials and have degree 0.
c. Simplify function first to find degree (or if x's are being multiplied, add their exponents).
d. Each power function $a_i x^i$ in this sum is called a **term.**
e. The constants $a_n, a_{n-1}, \ldots, a_0$ are called **coefficients.**
f. The term a_0 is called the **constant term**. The term with the highest power, $a_n x^n$, is called the **leading term.**
g. To write a polynomial in **standard form,** we arrange its terms from highest power to lowest power, going from left to right.

Ex: Determine which are polynomial functions and state the degree of those that are.

a. $f(x) = 6 - 2x + 3x^7$

$f(x) = 3x^7 - 2x + 6$

SEVENTH DEGREE

b. $f(x) = 0$

NO DEGREE

c. $f(x) = 3x - 2$

LINEAR

(1^{st} DEGREE)

d. $f(x) = 12$

CONSTANT

(DEGREE OF 0)

e. $f(x) = -2x^5(x+4)^2$

$-2x^5(x^2+8x+16)$

$-2x^7 - 16x^6 - 32x^5$

SEVENTH DEGREE

f. $f(x) = \dfrac{3x^2-4}{2x^3+6}$

NOT A POLYNOMIAL

(Fractional exponent)

g. $f(x) = \dfrac{3x^2-4}{3x^2-4}$

$f(x) = 1$

CONSTANT

(DEGREE OF 0)

h. $f(x) = 3x^{-4} + 7$

NOT A POLYNOMIAL

(Negative Exponent)

Graphs of Polynomial Functions –

1. Smooth – no sharp corners or cuts
2. Continuous – can be drawn without lifting pencil from paper

Zeros or Roots of Polynomials –

1. r is an x-intercept of the graph of f or $f(r) = 0$
2. $(x - r)$ is a factor of f

Ex: Find the polynomial of degree 3 whose zeros are 3, 2, and –3. Then graph it to verify your result.

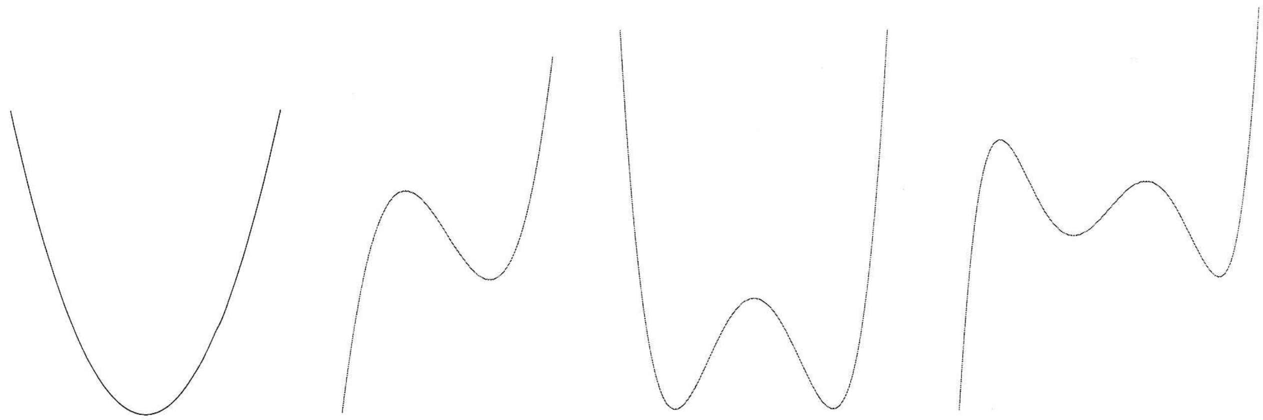
$f(x) = (x-3)(x-2)(x+3)$

$= (x-3)(x^2+x-6)$

$= x^3 + x^2 - 6x - 3x^2 - 3x + 18$

$f(x) = x^3 - 2x^2 - 9x + 18$

Like the power functions from which they are built, **polynomials** are defined for all values of x. Except for polynomials of degree zero (whose graphs are horizontal lines), the graphs of polynomials do not have horizontal or vertical asymptotes; they are smooth and unbroken. The shape of the graph depends on its degree; typical graphs are shown below.



Quadratic $n = 2$ | Cubic $n = 3$ | Quartic $n = 4$ | Quintic $n=5$

Turning Points – If f is a polynomial of degree n, then f has at most $n-1$ turning points (where the graph changes from an increasing to decreasing function or from a decreasing to an increasing function).

End Behavior – For large x-values (either positive or negative), the graph of f behaves like the graph of $y = a_n x^n$.

When viewed on a large enough scale, the graph of the polynomial $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdot \cdot \cdot +$ $a_1 x + a_0$ looks like the graph of the power function $y = a_n x^n$. This behavior is called the **long-run behavior** of the polynomial. Using limit notation, we write:

$$\lim_{x \to \infty} p(x) = \lim_{x \to \infty} a_n x^n$$

$$\lim_{x \to -\infty} p(x) = \lim_{x \to -\infty} a_n x^n$$

Ex: Given the polynomial $q(x) = 3x^6 - 2x^5 + 4x^2 - 1$, where $q(0) = -1$, is there a reason to expect a solution to the equation $q(x) = 0$? If not, explain why not. If so, how do you know?

Since the graph will look like $f(x) = 3x^6$ on the large scale and exhibit the same end behavior, it must have at least two solutions since $f(x) = 3x^6$ takes on large positive values as x grows large (either positive or negative). Since the graph is smooth and unbroken, it must cross the x-axis at least twice to get from $q(0) = -1$ to positive values it attains as x approaches negative and positive infinity.

Ex: For the polynomial $f(x) = x^2(x-2)(x+2)$ find the following:

a. The x-intercepts and y-intercepts of the graph.

x-int: $y=0$

$0 = x^2(x-2)(x+2)$

$x^2=0 \quad x-2=0 \quad x+2=0$

$\boxed{x=0 \quad x=2 \quad x=-2}$

y-int: $(x=0)$

$f(0) = 0^2(0-2)(0+2)$

$f(0) = 0$

$\boxed{(0,0)}$

b. Determine whether the graph crosses or touches the x-axis at each x-intercept.

IT WILL TOUCH X-AXIS AT $X=0$ AND CROSS AT $X=-2$ AND $X=2$.

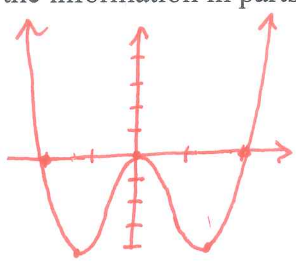
c. End behavior--find the power function that the graph of f resembles for large values of x.

$f(x) = x^4$

d. Use a graphing calculator to graph f.

e. Determine the number of turning points on the graph. Approximate the turning points to the nearest hundredth. $x=-1.41 \quad x=0 \quad x=1.41$

f. Use the information in parts a through e to sketch a graph of f by hand.



Ex: The volume, V, in milliliters, of 1 kg of water as a function of temperature T is given, for $0 \le T \le 30^\circ$ C by: $V = 999.87 - 0.06426T + 0.0085143T^2 - 0.0000679T^3$.

a.) Graph V and describe the shape of your graph. Does V increase or decrease as T increases? Does the graph curve upward or downward? What does the graph tell us about how the volume varies with temperature?

WINDOW:
$0 \le x \le 30$
$998 \le y \le 1004$

IF $0 \le y \le 1500$
HORIZONTAL LINE

TO SEE CUBIC
$-500 \le x \le 500$
$-3000 \le y \le 5000$

V decreases from $0 \le t \le 3.961$ and increases for $3.961 < t < 30$. The graph is concave up over the entire interval. The volume decreases at a decreasing rate from 0 to 3.961° and then increases at an increasing rate as the temperature rises.

b.) At what temperature does water have the maximum density? How does that appear on your graph? (Density = Mass/Volume. In this problem, the mass of the water is 1 kg.)

Density $= \frac{1}{\text{Volume}}$

Max density will be when the volume is the lowest, which is at 3.961°

HW: pg 445-447 #1,2,4-6,8-12,22,25-27,31